Testing for the Existence of a Generalized Wiener Process: the Case of Stock Prices

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ABSTRACT

In this article, we present two nonparametric trispectrum based tests for testing the hypothesis that an observed time series was generated by what we call a *generalized Wiener process* (GWP). Assuming the existence of a Weiner process for asset rates of return is critical to the Black-Scholes model and its extension by Merton (BSM). The Hinich trispectrum-based test of linearity and the trispectrum extension of the Hinich-Rothman bispectrum test for time reversibility are used to test the validity of BSM. We apply the tests to a selection of high frequency NYSE and Australian (ASX) stocks.

Keywords: Geometric Brownian Motion, Black-Scholes-Merton Model, Trispectrum, Linearity, Time Reversibility, Bonferroni Test.

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1 INTRODUCTION

Some years ago the Black-Scholes-Merton (BSM) model became a preferred tool for pricing and hedging options and other derivatives securities. The derivatives business now exceeds 415 trillion in the US$ market and has been implicated as a source of the 2008-9 global financial crisis.[1] Critical to both the Black-Scholes (1973) model and its extension by Merton (1973) is the assumed existence of a Weiner process for asset rates of return (Campbell, Lo and MacKinlay (1997), Gourieroux and Jasiak (2001)).

A Wiener process (also called Brownian Motion) is a continuous time Gaussian process with independent increments. To date, there has not been a rigorous testing procedure for checking the validity of the assumption that a Weiner process exists in the case of high frequency financial market data. Here, we present two nonparametric trispectrum-based tests of the hypothesis that an observed time series was generated by what we call a *generalized Wiener process* (GWP). We apply these tests to four high frequency NYSE stocks and a number of minute-minute Australian stocks chosen from the top 50 companies in terms of market capitalization.

The discrete-time version of a Wiener Process is a Gaussian random walk. Suppose that for the discrete times $t_n = n\tau$, $\left\{e\left(t_n\right)\right\}$ is a sequence of

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[1] See, for example, portfolio.com/news-markets/national-news/portfolio/ 2008/02/19/

non-Gaussian, identically and independently distributed random variables with finite moments, and $h(t_n) = h(-t_n)$ is a symmetric absolutely summable filter, band-limited at frequency f_o. We define the Generalized Wiener process (GWP) $\{x(t_n)\}$ by the following model of the first differences of the GWP:

$$\Delta x(t_n) = x(t_n) - x(t_{n-1}) = \sum_{k=-\infty}^{\infty} h(t_k) e(t_{n-k}), \tag{1.1}$$

where we use the discrete-time representation to avoid unnecessary continuous time stochastic calculus infinitesimals.

The term band-limited means that the complex transfer function $H(f) = \sum_{n=-\infty}^{\infty} h(t_n) \exp(-i2\pi f t_n)$ is zero for frequencies $|f| > f_o$. $H(f)$ is real since the filter in (1.1) is symmetric.[2] We assume that $H(f) > 0$ for $|f| \le f_o$.

If $\tau = 1/2f_o$ then the sampling rate is at the Nyquist frequency implying from the sampling theorem that the continuous time process is completely determined from the discrete-time values (Bracewell, 1978).

The first differences process defined in (1.1) is a linear and time reversible time series. It is time reversible since the joint distribution of

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[2] This is an important property because it ensures that the filter $h(t)$ is non-causal. Filtering using a causal filter can generate phase shifts that will generate a time irreversible probabilistic structure even when the source input time series [i.e. the $\{e(t_n)\}$] is a time reversible process.

$\left\{\Delta x\left(t_{n_1}\right), \Delta x\left(t_{n_2}\right), \cdots, \Delta x\left(t_{n_k}\right)\right\}$ is the same as that for the sequence

$\left\{\Delta x\left(t_{n_k}\right), \cdots, \Delta x\left(t_{n_2}\right), \Delta x\left(t_{n_1}\right)\right\}$.

The Hinich (1982) bispectrum-based test of linearity was employed by Hinich and Patterson (1989) to falsify the hypothesis that intraday stock returns are linear.[3] Hinich and Patterson (1995) confirmed that daily stock rates of return are also nonlinear. Yet the clear findings of these articles had little impact on the use of the BSM model to price financial options. This was probably due to the fact that neither article made a direct connection with the assumptions of the BSM model.

Note that the first differences of the GWP (1.1) is not a martingale difference process unless $h\left(t_k\right)=0$ for all $k \neq 0$. The strict form of the efficient market hypothesis assumes that the excess (unanticipated) stock returns is a martingale difference process. Hinich and Patterson (1992) use a test based on a modification of the bispectrum to reject the null hypothesis that the estimates of the daily GE excess returns follow a martingale difference hypothesis.

The trispectrum test of time reversibility that we apply here is built upon that of Hinich and Rothman (1998) and the linearity test is constructed from the aforementioned study by Hinich (1982). The time series data that is subject to test is observed first differences of the form

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[3] Also see Hinich (2008) for a demonstration of how the bispectrum time reversibility test can be used to falsify ARCH/GARCH models.

$y(t_n) = \Delta p(t_n)$. As long as the sample size of the observed time series is sufficiently large, we find that the trispectrum is a more powerful tool than the bispectrum since the former is a spectral decomposition of kurtosis while the latter is a spectral decomposition of skewness.[4] All distributions except for the Gaussian have non-zero kurtosis whereas all symmetric distributions have zero skewness.[5]

In Section 2, we briefly explain how the trispectrum is defined. In Section 3 we explain how the trispectrum can be estimated. In Sections 4 and 5 we, respectively, discuss how the time reversibility and linearity of a GWP can be tested.

In Section 6, we present a Bonferonni test framework that allows us to measure the pervasiveness of possible rejection patterns across valid tri-frequency combinations. In Section 7, we apply the tests in the context of high frequency stock rate of return data to see if the assumptions underlying BSM are valid. Section 8 contains some conclusions.

2 THE TRISPECTRUM

The trispectrum captures the values of the fourth order cumulant spectrum for four frequencies that sum to zero. Details of the definitions of the trispectrum and the fourth order cumulant spectrum and their

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[4] The skewness of $y(t_n)$ is $\gamma_{y3} = \sigma_y^{-3}\left[E\left(y(t_n)\right)\right]^3$ where σ_y^2 is the variance of $y(t_n)$

[5] The kurtosis is $\kappa_{y4} = \sigma_y^{-4}\left[E\left(y(t_n)\right)\right]^4 - 3$.

asymptotic properties are presented by Dalle Molle and Hinich (1995). For our purposes here we selected a simpler, yet mathematically correct, definition of the trispectrum $T_{yyyy}\left(f_{k_1}, f_{k_2}, f_{k_3}\right)$ of observed values of $y(t_n) = \Delta x(t_n)$ for a segment $\{y(0), y(t_1), y(t_2), \cdots, y(t_{L-1})\}$ of L successive observations and for the discrete (Fourier) frequencies $f_k = k/T$ where $T = L\tau$. To economize on notation we use the index k to denote the k-th Fourier frequency rather than f_k.

The trispectrum value at the triple frequency (k_1, k_2, k_3) is

$$T_{yyyy}(k_1, k_2, k_3) = \lim_{L\to\infty} E\left[\frac{Y(k_1)Y(k_2)Y(k_3)Y(k_4)}{L}\right] \tag{2.1}$$

where $Y(k) = \sum_{n=0}^{L-1} y(t_n)\exp(-i2\pi f_k t_n)$ is the discrete Fourier transform of the segment and $k_4 = L - k_1 - k_2 - k_3$ for the discrete frequencies in the principal domain, except for frequencies in the three linear (index) subsets $S_1 = \{k_1 + k_2 = 0 \ \& \ k_3 + k_4 = 0\}$, $S_2 = \{k_1 + k_3 = 0 \ \& \ k_2 + k_4 = 0\}$ and $S_3 = \{k_1 + k_4 = 0 \ \& \ k_2 + k_3 = 0\}$. We exclude these three sets to avoid the unnecessary complications of the trispectrum on these sets.

The limit exists from Theorem 4.3.1 in Brillinger (1981) because the linear model in (1.1) is strictly stationary and satisfies the Brillinger mixing condition 2.6.1. The stationarity and the mixing condition are necessary to obtain the asymptotic properties of the trispectrum estimates presented in the next section.

The principal domain Ω, defined within the cube given by the indices $\{-k_o < k_1 < k_o,\ -k_o < k_2 < k_o,\ -k_o < k_3 < k_o\}$ where $k_o = f_o T$, is given by $\Omega = \Omega_+ \cup \Omega_- - (S_1 \cup S_2 \cup S_3)$, where

$$\Omega_+ = \{1 \le k_1 \le k_o,\ 1 \le k_2 \le k_o,\ 1 \le k_3 \le \min(k_2, k_o - k_1 - k_2)\ \&\ k_1 + k_2 < k_o\} \quad (2.2)$$

$$\Omega_- = \{1 \le k_1 \le k_o,\ 1 \le k_2 \le k_o,\ k_o - k_1 - k_2 \le k_3 \le -k_2\}. \quad (2.3)$$

The normalized trispectrum is defined to be

$$\mathrm{K}_{yyyy}(k_1,k_2,k_3) = \frac{T_{yyyy}(k_1,k_2,k_3)}{\sqrt{S_y(k_1)S_y(k_2)S_y(k_3)S_y(k_4)}} \quad (2.4)$$

where $S_y(k)$ is the spectrum of $\{y(t_n)\}$ and $k_4 = L - k_1 - k_2 - k_3$. For the linear process defined in (1.1) the trispectrum is $T_{yyyy}(k_1,k_2,k_3) = \sigma_y^4 \kappa_{y4} H(k_1)H(k_2)H(k_3)H(k_4)$ and $S(k) = \sigma_y^2 H^2(k)$ where κ_{y4} is the kurtosis and σ_y^2 is the variance of $y(t_n)$. Thus, for this linear process, the normalized kurtosis values are $\mathrm{K}_{yyyy}(k_1,k_2,k_3) = \kappa_{y4}$, a real number for each triple index.

3 ESTIMATING THE TRISPECTRUM

Suppose that we observe the process $\{y(t_n)\}$ for a time period of length $N\tau$ and divide this period into M non-overlapping frames each of length L, assuming for simplicity that N is divisible by L. The estimate of the trispectrum for each frequency triple in the principal domain, but not in the three linear subsets given above, is the frame averaged estimate given in Dalle Molle and Hinich (1995)

$$\hat{T}_{yyyy}(k_1,k_2,k_3) = \frac{1}{M}\sum_{m=1}^{M}\frac{Y_m(k_1)Y_m(k_2)Y_m(k_3)Y_m(L-k_1-k_2-k_3)}{L} \qquad (3.1)$$

where $Y_m(k)$ is the k-th term of the discrete Fourier transform of the m-th frame. Using the frame averaged estimate of the spectrum

$$\hat{S}_y(k) = \frac{1}{M}\sum_{m=1}^{M}\frac{|Y_m(k)|^2}{L} \qquad (3.2)$$

then the sample normalized trispectrum is given by

$$\hat{\mathrm{K}}_{yyyy}(k_1,k_2,k_3) = \left[\hat{S}_y(k_1)\hat{S}_y(k_2)\hat{S}_y(k_3)\hat{S}_y(L-k_1-k_2-k_3)\right]^{-1/2}\hat{T}_{yyyy}(k_1,k_2,k_3).$$

(3.3)

Let $L = N^c$ then $M = N^{(1-c)}$, where the bandwidth frame coefficient c is in the interval $0 < c < 1/3$. Dalle Molle and Hinich (1995) applied the asymptotic results in Brillinger and Rosenblatt (1967) to get the following large sample results:

For the set of indices (k_1,k_2,k_3) in the principal domain, except for the three linear subsets, $\mathrm{Re}\left(\hat{T}_{yyyy}(k_1,k_2,k_3) - T_{yyyy}(k_1,k_2,k_3)\right)$ and $\mathrm{Im}\left(\hat{T}_{yyyy}(k_1,k_2,k_3) - T_{yyyy}(k_1,k_2,k_3)\right)$ are approximately jointly normally distributed with zero means and variances $\left(N^{(1-3c)}/2\right)S_y(k_1)S_y(k_2)S_y(k_3)S_y(k_4)$ for $k_4 = L-k_1-k_2-k_3$, and zero covariances across the frequency triples and between the real and imaginary parts of the trispectrum values. Thus, the real and imaginary parts of the normalized trispectrum values are approximately normal with variance ½.

4 TESTING TIME REVERSIBILITY OF THE GWP

Assume that the observed values of $y(t_n) = \Delta x(t_n)$ are generated by a GWP. Since it is time reversible we can use the trispectrum version of the Hinich-Rothman test. Let $\hat{\mathrm{K}}_{yyyy}(k_1,k_2,k_3)$ denote the estimate of the normalized trispectrum value where the spectral estimates are substituted for the true spectral values in the expression for the variance of the trispectral estimates. Under this null hypothesis the large sample distribution of $2N^{(1-3c)}\left|\mathrm{Im}\,\hat{\mathrm{K}}_{yyyy}(k_1,k_2,k_3)\right|^2$ is $\chi_1^2(0)$, a central chi square with one degree-of-freedom. Since the trispectrum values are approximately independently distributed over the triple frequencies in Ω, the statistic

$$R = \sum_{(k_1,k_2,k_3)\in\Omega} 2N^{(1-3c)}\left|\mathrm{Im}\,\hat{\mathrm{K}}_{yyyy}(k_1,k_2,k_3)\right|^2 \tag{4.1}$$

has a central chi square $\chi_D^2(0)$ distribution where D is the number of triple frequencies in Ω.

Let $F(r\,|\,D,0)$ denote the cumulative distribution function of a $\chi_D^2(0)$ random variable. Let $U_R = F(R\,|\,D,0)$ denote transforming the statistic R

into a uniform (0,1) variate under the null hypothesis of time reversibility. The trispectrum program computes the p-values given by $\left[1-U_R\left(k_1,k_2,k_3\right)\right]$ for all the triple frequencies in the principal domain.

5 TESTING LINEARITY OF A GWP

Under the null hypothesis that the observed values of $y\left(t_n\right)=\Delta x\left(t_n\right)$ are generated by a GWP, the large sample distribution of $W\left(k_1,k_2,k_3\right)=N^{(1-3c)}\left|\hat{\mathrm{K}}_{yyyy}\left(k_1,k_2,k_3\right)\right|^2$ is $\chi_2^2\left(\lambda\right)$, a non-central chi square with two degree-of-freedoms and a non-central parameter $\lambda=\kappa_{y4}^2$. Let $F\left(r\,|\,2,\lambda\right)$ denote the cumulative distribution function of a $\chi_2^2\left(\lambda\right)$ random variable. Let $U_L\left(k_1,k_2,k_3\right)=F\left(W\left(k_1,k_2,k_3\right)|\,2,\lambda\right)$ denote transforming the statistic for the scaled square magnitude of the normalized trispectrum value to a uniform (0,1) distribution under the null hypothesis of linearity. The estimate of the non-centrality parameter is the mean value of the $W\left(k_1,k_2,k_3\right)$'s in Ω, which are approximately independently distributed in the principal domain. The trispectrum program computes the cumulative density function for $U_L\left(k_1,k_2,k_3\right)$ for all combinations of $\left(k_1,k_2,k_3\right)\in\Omega$ and then calculates the value $U_L^q\left(k_1,k_2,k_3\right)$ associated with a user specified quantile value q such that $\left(0\le q\le 1\right)$ and with associated p-value given by $\left[1-U_L^q\left(k_1,k_2,k_3\right)\right]$ associated with a $\left(1-q\right)$ level of significance.[6]

6 A BONFERRONI TESTING METHODOLOGY TO GAUGE THE PERVASIVENESS OF REJECTION PATTERNS ACROSS VALID TRI-FREQUENCY COMBINATIONS

In order to gauge how pervasive the rejection patterns of the above test statistics are across valid tri-frequencies in the principal domain, we also employ a testing procedure based on the Classical Bonferroni procedure. The statistical methodology underpinning the Bonferroni test is the Union-Intersection (UI) method proposed by Roy (1953) that can be used, in principle, to construct alternative joint tests of both the linearity and time reversibility hypotheses developed above.[7]

The (UI) procedure involves the construction of a joint hypothesis test from a series of individual hypothesis tests. Suppose we have a number of individual hypotheses of the form

$$\begin{matrix} H_{0_k} = TS_k \\ H_{1_k} \neq TS_k \end{matrix}, \text{ for } k=\left\{1,2,\ldots,D\right\}. \tag{6.1}$$

In the current context, the individual tests refer to those that are involved in the summation process in (4.1) producing the CUSUM time

[6] Note that in this article we adopt a value for quantile q of 0.9 for the four NYSE stocks and a value of 0.99 for the Australian stocks producing levels of significance of 10% and 1% respectively.

[7] Also see Miller (1966), Savin (1980, 1984), and Hochberg and Tamhane (1987).

reversibility test statistic R or each $W(k_1,k_2,k_3)$ associated with the linearity test where the number of individual tests equal *D,* the number of triple frequencies in the principal domain Ω.

The (UI) procedure involves combining the individual hypotheses to form an overall joint hypothesis H_0, defined as an intersection of a family of individual hypotheses. In the current context, this is given by

$$H_0 = \bigcap_{k=1}^{D} H_{0_k} , \tag{6.2}$$

where D is the total number of tri-frequency tests to be performed. Note that H_0 is interpreted as a 'global' null in which all H_{0_k} are true against the alternative H_1 which is interpreted as 'not H_0', (Hochberg and Tamhane (1987, pp. 28-29), Rom (1990, p. 633), and Samuel-Cahn (1996, p. 932)).

The rejection region for H_0 corresponds to the union of rejection regions for the individual hypotheses H_{0_k}, giving $H_1 = \bigcup_{k=1}^{D} H_{1_k}$. This means that H_0 is rejected if and only if at least one of the H_{0_k}'s is rejected (Hochberg and Tamhane, 1987, p. 3, 7, 28).

Given the individual tri-frequency tests making up (4.1) or each of the $W(k_1,k_2,k_3)$'s for $(1 \le k \le D)$ that underpin the individual tests of H_{0_k} versus H_{1_k}, we can express the resulting rejection region for each test as

$$TS_k > \xi_k \ , \qquad (1 \le k \le D) \tag{6.3}$$

with the critical constants ξ_k constrained so that the rejection region of the (UI) test of H_0 is the union of rejection regions of (6.3). For the rejection region of the joint test to have size α, the ξ_k's must satisfy

$$\Pr_{H_0}\{TS_k > \xi_k \ \text{ for some } k = 1,2,...,D\} = \alpha \ . \tag{6.4}$$

We can simplify (6-3)-(6.4) further by assuming that the 'D' testing problems are to be treated symmetrically with regard to the relative importance attached to Type I and Type II errors, thereby implying that the marginal levels $\Pr_{H_{0k}}\{TS_k > \xi_k\}$ should be the same for $k = 1,2,...,D$. Moreover, the TS_k's have the same marginal distribution under H_{0_k}, corresponding to either a $\chi_1^2(0)$ or $\chi_2^2(\lambda)$ distribution, respectively. We can therefore set the $\xi_k's$ to the same value (Hochberg and Tamhane, 1987, pp. 29-30). Thus, after setting $\xi_k = \xi,$ for all k in (6.3), the (UI) test will reject if

$$\max_{1 \le k \le D} \ TS_k > \xi \ , \tag{6.5}$$

with ξ being chosen so that

$$\Pr_{H_0}\{\max_{1 \le k \le D} \ TS_k > \xi\} = \alpha \ , \tag{6.6}$$

(Hochberg and Tamhane (1987, p. 30)).

In this article, we use the Classical Bonferroni inequality to enumerate (6.6). This involves using the inequality

$$\Pr\left\{\bigcup_{i=1}^{D}\left(P_i \le \alpha / D\right)\right\} \le \alpha \qquad \left(0 \le \alpha \le 1\right), \tag{6.7}$$

which guarantees that the probability of rejecting at least one hypothesis when all are true is not greater than α (Simes, 1986, p. 751). This procedure leads to a critical value for ξ given by $\left(\alpha / D\right)$ where α is a user specified level of significance and D is the total number of tri-frequency linearity or time reversibility tests as defined above. In general, the testing procedure leads to the rejection of the joint null hypothesis H_0 if $P_1 \le \left(\alpha / D\right)$, where P_1 is the smallest p-value associated with one of the tri-frequency tests. In this article, we set $\alpha = 0.1,\ 0.05,\ \textit{and}\ 0.01,$ signifying ten percent, five percent and one percent levels of significance, respectively.

We implement the Bonferroni test procedure and calculate the number of individual tri-frequency tests that would have led to the rejection of the joint null of linearity or time reversibility according to the Classical Bonferroni test procedure outlined above. We express these rejections as a percentage of the total number of p-values (or tests) that were admissible and subsequently performed. The larger this percentage value is the more pervasive will be the rejection of the null hypotheses of linearity and time reversibility across admissible tri-frequency combinations.

7. APPLICATION TO STOCK RATES OF RETURN

We apply the two (i.e. linearity and time reversibility) tests to test the validity of the GWP in explaining the stochastic properties of the rates of return for a number of stocks listed on the New York Stock Exchange (NYSE) and to an assortment of companies that are in the top 50, in terms of market capitalization, listed on the Australian Stock Exchange. Our results can be considered as a generalization of the statistical analysis by Lo and MacKinlay (1988).

Recall from the discussion in Section 1 that the Black-Scholes-Merton model for pricing options assumes that the rates of return for the stock in an option follow a Weiner process. We will allow a hypothetical generalization of the stochastic model of the returns to be a GWP as defined by expression (1.1). The rate of return for a stock at time t_n is $\Delta p(t_n) = \log p(t_n) - \log p(t_{n-1})$ where $p(t_n)$ is the price of the stock at time t_n. Then it follows that $y(t_n) = \Delta p(t_n)$ in the trispectrum definition in Section 2.

We first apply the two trispectrum based tests to four NYSE intraday rates of return time series for the period Jan 4, 1999 to Dec 29, 2000. This is an interesting period because it was the peak of the 'dot com' bubble. The stocks are Birmingham Steel (BIR), EOG Resources (EOG), First Energy (FE), and Imation (IMN).

All original tick strike prices were adjusted for dividends and stock splits.[8] There are 36 such aggregated rates for each trading day of six hours yielding 19,656 returns. The summary statistics of these four returns are presented in Table 1.[9] We used a frame length of 18 ten minute returns or a half trading day which yields a bandwidth frame coefficient of $c = 0.29$. There are 73 trispectral estimates in the principal domain for this particular frame length.

[INSERT TABLE 1 ABOUT HERE]

Examination of the summary statistics for the four NYSE stocks in Table 1 indicates sizeable kurtosis values ranging from 14.6 for EOG to 102.0 for IMN. Furthermore, the empirical distribution of three stocks (e.g. BIR, FE and IMN) are negatively (left) skewed while EOG is positively (right) skewed. The kurtosis values in this range point to the presence of fat tails in the empirical distribution function of the four NYSE stock rates of return, and together with the skewness results, point to substantial deviations from the Gaussian distribution. However, the large

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[8] No adjustments were actually required for the four NYSE stocks. However, some adjustments were required for the Australian stocks, notably for BHP-Billiton, Brambles, Coca-Cola Amatil and Lend lease, the details of which will be addressed later on in this section – also see Table 5.

[9] We use the term 'sigma' rather than standard deviation to reduce width of the standard deviation column.

sample convergence properties of the trispectral estimates are highly problematical for such fat tailed time series, in particular. We have therefore trimmed the rates of return time series in order to improve the validity of the use of the asymptotic properties of the trispectral estimates and tests.

Trimming data to make sample means less sensitive to outliers has been used in applied statistics for many years. Trimming is a simple data transformation that makes statistics based on the trimmed sample more normally distributed. To trim the upper and lower $\omega/2$ % values of the sample $\{x(t_1),...,x(t_N)\}$ we order the data and compute the $\omega/200$ quantile $x_{\omega/200}$ and the $1-\omega/200$ quantile $x_{1-\omega/200}$ of the order statistics. We then set all sample values less than the $\omega/200$ quantile to $x_{\omega/200}$ and all sample values greater than the $1-\omega/200$ quantile to $x_{1-\omega/200}$. The remaining $(100-\omega)\%$ data values are left unchanged.

We found for the NYSE data that we obtained good results from trimming the top 98% and bottom 2% of the returns $(i.e.\ \omega = 4\%)$. The descriptive statistics of the trimmed NYSE rates of return are presented in Table 2. It is apparent from inspection of this table that the skewness and kurtosis values have been significantly reduced compared to those listed in Table 1, together with the difference between the maximum and minimum values. The time reversibility and nonlinearity tests were applied to the trimmed returns data and the results are summarized in the first four rows of Table 10. Inspection of Table 10 shows that the hypothesis of time reversibility is strongly rejected for all four stocks. The results of the linearity test is more mixed with the test being rejected strongly for IMN, rejected at the 1 percent level of significance for BIR, at the 5 percent level of significance for FE and being rejected at the 10 percent level of significance for EOG.[10]

[INSERT TABLE 2 ABOUT HERE]

The results from applying the Bonferroni test procedure to the four stocks is listed in Table 3 and Table 4 for the linearity and time reversibility tests. The critical values for the Bonferroni tests at 10, 5 and 1 per cent levels of significance were determined to be 0.00136986, 0.00068493 and 0.00013699, respectively. In principle, if one p-value is less than or equal to these values, we secure rejection of the joint null hypotheses of linearity and time reversibility at the relevant levels of significance. Inspection of these tables indicate that more than one individual test secures this type of rejection. In relation to the linearity test (i.e. Table 3), for example, BIR secures the most rejections with 13.70%, 12.33% and 10.96% of all individual tests in the principal domain producing p-values that are less than or equal to the relevant Bonferroni critical value cited above. The percentages in the two tables

[10] Note that for the four NYSE stocks, the linearity test was assessed at the 90% quantile value.

indicate that, for all stocks and tests, the null hypotheses of linearity and time reversibility are rejected by the Bonferroni test procedure with the largest percentage of rejections for both tests being recorded by the BIR stock, followed by IMN, EOG and then FE. Note also that a higher percentage of rejections are secured for the time reversibility test (Table 4) than for the linearity test (Table 3) which is consistent with the nature of rejections cited in Table 10 for these four particular stocks.

[INSERT TABLES 3 AND 4 ABOUT HERE]

To complement the US stock data we obtained twenty-three large cap Australian stocks traded on the ASX (Australian Securities Exchange).[11] The ASX is the primary stock exchange in Australia and is an all-electronic exchange. The ASX has a pre-market session occurring from 07:00am to 10:00am and the normal trading session occurs over the time interval 10:00am to 04:00pm. The trading session officially closes at 4:00pm and between 4:00pm and 4:10pm the market is once again placed in 'pre-open' phase. Under the current electronic trading system, market closing prices are determined at a random time within the time interval 4:10pm and 4:12pm using a single price auction.

[11] Note that useful information about the electronic market can be found at the following web addresses: http://en.wikipedia.org/wiki/Australian_Securities_Exchange, and http://www.asx.com.au/resources/education/basics/price_calculations.htm.

The selected companies are chosen from the top 50 companies in terms of market capitalization and encompass a good coverage of the main industrial sectors of the market. The specific details of these companies are listed in Table 5. Note that adjustments were made to the original spot price data to correct for a number of stock splits that are identified in Table 5.

[INSERT TABLE 5 ABOUT HERE]

The data were provided by the industry and academic research center 'Securities Industry Research Centre of Asia-Pacific' Limited (SIRCA).[12] The data for each stock are minute-by-minute time series of spot price data over the time interval 10:00:00 to 16:06:00 for the period January 2, 1996 to December 30, 2005.[13] We calculated rates of return from these spot price data. The spot price concept underpinning the rate of returns calculation for the Australian stocks can be interpreted as a time weighted mid-point price that is calculated by averaging time weighted asks and bids over the time segments that together constitute a time interval of a minute. The scope and number of time segments arising within a minute's time interval is ultimately determined by the tick frequency of the stock transactions occurring within each minute. It should also be noted that, by averaging over the 'Asks' and 'Bids', the

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[12] The SIRCA web address is: http://www.sirca.org.au/.

[13] Note that under the old electronic trading platform, the market closing phase occurred between 16:04:00 and 16:06:00.

spot prices dynamics are not unduly influenced by micro-structure phenomenon such as 'Bid-Ask Bounce'.

The summary statistics of the ASX Australian stock returns are listed in Table 6. The most noticeable result is that these high frequency rates of return are very fat tailed, with kurtosis values ranging from a low of 571.0 (for CCL) to a high of 7830.0 (for BHP). These values are an order of magnitude higher than the comparable results for the four NYSE stock returns cited in Table 1. Moreover, around 70% of the returns are negatively (left) skewed. These results suggest that the empirical distribution functions of Australian returns deviate significantly from the Gaussian distribution.

[INSERT TABLE 6 ABOUT HERE]

We found that we obtained robust results by trimming the top 95% and bottom 5% of the returns ($\omega = 10\%$).[14] The statistics for the trimmed Australian returns are presented in Table 7. As expected, the skewness and kurtosis values of the trimmed returns are reduced significantly toward the Gaussian zero value (e.g. compare columns 4 and 5 of Table 7 with those of Table 6) but the p-values for the Hinich trispectrum based test of Gaussianity for the trimmed returns data are all less than or equal to 0.1E-05 indicating that the empirical distribution function of the

[14] In this particular case, the large sample approximation to the asymptotic theory is aided by both the trimming operations as well as the large number of sample frames over which frame averaging subsequently occurs.

trimmed returns data are still non-Gaussian.[15] It should also be noted that the trimming operation has significantly reduced the range of the returns data – for example, compare the last two columns of Table 7 with those of Table 6.

[INSERT TABLE 7 ABOUT HERE]

For the Australian returns we used a frame length of 60 one minute returns which yields a bandwidth frame coefficient of $c = 0.3$. The overall sample size is 927,776 observations and with a frame length of 60 one minute returns, produces frame averaging over 15,462 frames. There are also 2,920 trispectral estimates in the principal domain for this frame length.

The results for the time reversibility and linearity tests are cited in Table 10 (rows 5 to 27). Both time reversibility and linearity are strongly rejected by these tests for all of the Australian stock returns – in all cases, the p-values are less than 0.1E-05. Also note that, because of the larger sample sizes involved, the linearity test was computed at the 99% quantile for the Australian returns.

The results from the application of the Bonferroni test procedure for the linearity and time reversibility tests are documented in Table 8 and Table 9, respectively. Note from inspection of both tables that a significant number of individual p-values are less than the associated Bonferroni critical values of 0.00003425, 0.00001712 and 0.00000342 for the 10%,

[15] These results are available from the authors upon request.

5% and 1% levels of significance. For example, in Table 8, for AGL, the application of the '99% quantile' linearity test resulted in 13.84%, 13.49% and 13.05% of all the tri-frequencies in the principal domain having p-values less than or equal to the above 10%, 5%, and 1% Bonferroni critical values, respectively. Similarly, in Table 9, for BHP, the application of the time reversibility test resulted in 10.07%, 8.97% and 6.75% of all the tri-frequencies in the principal domain having p-values less than or equal to the above 10%, 5%, and 1% Bonferroni critical values. Recall that according to the Classical Bonferroni procedure, it only requires one p-value to be less than or equal to the above Bonferroni critical values to secure rejection of the joint null hypotheses of linearity or time reversibility at the above associated levels of significance, according to the (UI) test methodology.

[INSERT TABLES 8 AND 9 ABOUT HERE]

Note that the critical values applicable for the Australian returns data are much smaller in magnitude than the corresponding critical values applicable to the NYSE data that were listed in Tables 3 and 4. This follows because of the larger number of trispectral estimates in the principal domain (and total number of individual tests to be performed) in the case of the Australian data. Essentially, because of the larger number of individual tests involved and the higher probability of random chance leading to the false rejection of the null hypothesis when it is true, smaller critical values are needed to ensure that the probability of falsely rejecting the null hypothesis when it is true is not greater than 10%, 5% and 1%, respectively.

The range of percentage results for the linearity test is documented in Table 8. They range from a low for NAB of (6.30%, 5.82%, 5.72%) to a high for SGP of (20.14%, 19.76%, 19.52%). The percentage results for time reversibility in Table 9 range from a low for ANZ of (6.27%, 5.82% and 4.62%) to a high for BIL of (25.58%, 24.04%, 21.16%). As a group, the banking stocks (ANZ, CBA, NAB, SGB and WBC) appear to have the lowest percentage rejection levels for both the linearity and time reversibility tests. Other highly traded stocks such as BHP also have lower relative percentage rejection levels when compared to other stocks. However, for most other industrial groups, the evidence is broadly mixed with the group containing stocks with relatively low and high percentage rejection levels.

[INSERT TABLE 10 ABOUT HERE]

However, in all cases, the conventional CUSUM and Bonferroni test procedures demonstrate, for all of the Australian stocks, that the hypotheses of linearity and time reversibility are strongly rejected for the returns data. Thus, the data does not provide statistical support for the Generalized Weiner Process as an acceptable model of stock price returns as required in the Black-Scholes-Merton model of options pricing. Moreover, because of the trimming we have employed, the time reversibility and nonlinearity in the returns cannot be attributed to the presence of outliers.

8. CONCLUSIONS

In this article, we have presented two nonparametric trispectrum based tests that are capable of testing the hypothesis that an observed time series was generated by a *generalized Wiener process* (GWP). The presence of a Weiner process is a critical assumption made about asset price dynamics (returns) in the Black-Scholes (1973) model and its extension by Merton (1973) – the central model that has been used to model the pricing of options in the finance literature and in risk management in the finance industry.

The two key assumptions of the GWP that we have tested are those of linearity and time reversibility. The trispectrum linearity test, which is an extension of the Hinich (1982) bispectrum test of linearity, is designed to test for linearity in intra-daily stock rates of return. The trispectrum time reversibility test is an extension of the Hinich-Rothman (1998) bispectrum test for time reversibility and is used to test for time reversibility of intra-daily stock returns.

We applied the two trispectrum based tests to four high frequency NYSE stocks and a number of minute-to-minute Australian stocks selected from the top 50 companies in terms of market capitalization. The tests showed that the returns were nonlinear and were not time reversible. Since the GWP is linear and time reversible, it was rejected by the tests and not supported by the data.

All of the returns data displayed significant evidence of kurtosis which posed problems for the large sample convergence properties of the trispectral estimates. Trimming was therefore employed in order to improve the validity of the asymptotic properties of the trispectral estimates and tests. While, as might be expected, the trimming operations moved the skewness and kurtosis values significantly closer to their Gaussian ideals, the empirical distribution function of the trimmed returns data was still found to be non-Gaussian and the tests applied to the these data confirmed that the data generating mechanism of the returns was both nonlinear and time-irreversible.

A Bonferroni test procedure was also used to investigate the pervasiveness of the rejection patterns. The Classical Bonferroni inequality was used to enumerate the test procedure and ensure that the probability of TYPE I error was strongly controlled at 10%, 5% and 1% levels of significance. While rejection of the joint null hypotheses of linearity or time reversibility according to this test procedure only required one p-value from a marginal (i.e. individual tri-frequency) test to be less than or equal to the relevant Bonferroni critical value, the empirical results indicated that many individual tests fulfilled this criterion. Specifically, the overall rejection percentages ranged broadly from between 7% to 30% of the total number of admissible tests in the principal domain. As such, the rejections were not secured by a small number of individual test outcomes driven by outliers in the returns data. The scope of the rejections, instead, point to significant statistical structure that caused the rejection of the GWP. This is supported further by the use of trimming which has the effect of controlling for the presence of outliers in the data. As such, the rejection of the hypotheses of linearity and time reversibility was not driven by the presence of outliers in the data.

These results suggest that a number of prominent contributions to the finance literature, employing the GWP assumption, will have to be reassessed and re-interpreted. Also, there are important implications for those in the finance industry who build statistical models of asset prices to assist in the pricing of derivatives. We have gone through a period of crisis where the Black-Scholes-Merton methodology for pricing options has clearly been found to be wanting. So, in future, we shall need to exercise much greater statistical care in understanding the properties of the time series that we use.

REFERENCES

Black, F., and Scholes, M. (1973), "The Pricing of Options & Corporate Liabilities", *Journal of Political Economy*, 81, 637-59.

Bracewell, R.N. (1978), *The Fourier Transform and its Applications*, Second Edition, New York: McGraw-Hill.

Brillinger, D. R. and Rosenblatt, M. (1967), "Asymptotic Theory of k-th Order Spectra," in, Harris, B. ed., *Spectral Analysis of Time Series*, New York: Wiley, 153-158.

Brillinger, D. R. (1981), *Time Series: Data Analysis and Theory, Extended Version.* New York, Holt, Rinehart and Winston.

Campbell, J. Y., Lo, A. W. and MacKinlay, A.C. (1997), *The Econometrics of Financial Markets*, Princeton, N.J., Princeton University Press, Section 9.1-9.2.

Dalle Molle, J.W. and Hinich, M.J. (1995), "Trispectral analysis of stationary random time series," *Journal of the Acoustical Society of America,* 97, 2963-2978.

Gourieroux, C. and Jasiak, J. (2001), *Financial Econometrics*, Princeton, N.J., Princeton University Press, Chapter 11.

Hinich, M.J. (1982), "Testing for Gaussianity and Linearity of a Stationary Time Series," *Journal of Time Series Analysis,* 3, 169-176.

Hinich, M.J. (2008), "Falsifying ARCH/GARCH Models using Bispectral Based Tests," *Communication in Statistics* – Theory and Methods, 38, 529-541.

Hinich, M.J. and Patterson, D.M. (1985), “Evidence of Nonlinearity in Daily Stock Returns, “*Journal of Business and Economic Statistics,* 3, 69-77.

Hinich, M.J and Patterson, D.M. (1989), "Evidence of Nonlinearity in the Trade-by-Trade Stock Market Return Generating Process," in Barnett, W., Geweke, J. and Shell, K. ed., *Economic Complexity, Chaos, Sunspots, Bubbles, and Nonlinearity*, New York: Cambridge University Press, Chapter 16.

Hinich, M.J and Patterson, D.M. (1992), "A New Diagnostic Test of Model Inadequacy which uses the Martingale Difference Criterion", *Journal of Time Series Analysis*, 13, 233-252.

Hinich, M.J and Rothman, P. (1998), "Frequency-Domain Test of Time Reversibly," *Macroeconomic Dynamics*, 2, 72-88.

Hochberg, Y., and Tamhane, A.C. (1987), *Multiple Comparison Procedures*. New York, John Wiley & Sons.

Lo, A. and MacKinlay, A.C. (1988), “Stock Market Prices do not Follow Random Walks: Evidence from a Simple Specification Test,” *Review of Financial Studies,* 1, 41-66.

Merton, R. C. (1973), "Theory of Rational Option Pricing". *Bell Journal of Economics and Management Science,* 4, 141–183.

Miller, R.G. (1966), *Simultaneous Statistical Inference, First Edition*. New York, McGraw-Hill.

portfolio.com/news-markets/national-news/portfolio/2008/02/19/.

Rom, D.M. (1990), "A sequentially rejective test procedure based on a modified Bonferroni inequality," *Biometrika*, 77, 663-665.

Roy, S.N. (1953), "On a heuristic method of test construction and its use in multivariate analysis," *Annals of Mathematical Statistics*, 24, 220-238.

Samuel-Cahn, E. (1996), "Is the Simes improved Bonferroni procedure conservative?" *Biometrika*, 83, 928-933.

Savin, N.E. (1980), "The Bonferroni and the Scheffe Multiple Comparison Procedures," *Review of Economic Studies*, XLVII, 255-273.

Savin, N.E. (1984): "Multiple Hypothesis Testing," in Griliches, Z., and Intriligator, M.D. ed., *Handbook of Econometrics, Vol. II*, New York and Oxford: North-Holland, 827-879.

Simes, R.J. (1986), "An improved Bonferroni procedure for multiple tests of significance," *Biometrika*, 73, 751-754.

Table 1 Descriptive Statistics of the Four NYSE Returns

	Mean	Sigma	Skew	Kurtosis	Max	Min
BIR	-0.758E-02	1.390	-0.35	34.4	21.21	-27.60
EOG	0.592E-02	0.521	0.70	14.6	9.28	-6.23
FE	-0.197E-03	0.311	-0.76	33.7	4.26	-8.49
IMN	-0.544E-03	0.467	-2.51	102.0	6.06	-16.10

Table 2 Descriptive Statistics after 4% Trimming of the Four NYSE Returns

	Mean	Sigma	Skew	Kurtosis	Max	Min
BIR	-0.850E-02	1.030	0.028	3.72	3.28	-3.28
EOG	0.290E-02	0.436	0.189	1.11	1.22	-1.11
FE	-0.374E-03	0.271	0.078	0.26	0.68	-0.65
IMN	0.420E-03	0.356	0.124	1.90	1.05	-1.00

Table 3 Classical Bonferroni (BF) '90 Percent Quantile' Linearity Test Statistics for the NYSE Returns (73 Trispectral Values)

Stocks	10% BF - % of p-values in PD < 0.00136986	5% BF - % of p-values in PD < 0.00068493	1% BF - % of p-values in PD < 0.00013699
BIR	13.70%	12.33%	10.96%
EOG	5.48%	5.48%	4.11%
FE	5.48%	1.37%	1.37%
IMN	6.85%	6.85%	5.48%

Table 4 Classical Bonferroni (BF) Time Reversibility Test Statistics for the NYSE Returns (73 Trispectral Values)

Stocks	10% BF - % of p-values in PD < 0.00136986	5% BF - % of p-values in PD < 0.00068493	1% BF - % of p-values in PD < 0.00013699
BIR	21.92%	17.81%	9.59%
EOG	5.48%	5.48%	5.48%
FE	2.74%	2.74%	2.74%
IMN	13.70%	9.59%	6.85%

Table 5 Details of Australian Companies

ASX Code	Name	Core Activities	Stock/Dividend Split Details
AGL	Australian Gas and Lighting Ltd	Retail Energy and Fuel	N.A.
AMC	Amcor Ltd	Packaging	N.A.
ANZ	Australian and New Zealand Banking Group Ltd	Banking and Financial Services	N.A.
BHP	BHP-Billiton Ltd	Mineral and Hydrocarbon Exploration, Production and Processing	BHP-Billiton merger on 29/6/2001 (from $21.418 to $10.495 at 10:00:00 on 29/6/2001).
BIL	Brambles Ltd	Commercial and Business Support (e.g. Packaging, Containers, Storage)	merger with UK services group GFN on 8/8/2001 (from $45.679 to $10.703 at 10:00:00 on 8.8.2001)
CBA	Commonwealth Bank of Australia	Banking and Financial	N.A.

	Ltd	Services	
CCL	Coca-Cola Amatil Ltd	Beverage Manufacturing, Marketing and Distribution	demerging of European operations on 13/7/1998 (from $11.29 to $7.125 at 10:00:00 on 13/7/1998)
CSL	CSL Ltd	Development, Manufacturing and Marketing of Pharmaceutical and Diagnostic products	N.A.
FGL	Fosters Group Ltd	Production and Marketing of Alcoholic Beverage	N.A.
FXJ	Fairfax Media Ltd	Newspaper, Magazines and Electronic Media Publishing	N.A.
LLC	Lend Lease Ltd	Retail Property and Asset Management and Development and Construction	1 to 1 bonus share offer in December 1998 (from $38.675 to $19.983 at 10:00:00 on 1/12/1998)
NAB	National Australia Bank Ltd	Banking and Financial Services	N.A.
NCM	Newcrest Mining Ltd	Exploration, Development, Mining and Sale of Gold	N.A.
QAN	Qantas Airways Ltd	Domestic and International Air Transportation Services	N.A.
QBE	QBE Insurance Group Ltd	Insurance and Financial Services	N.A.
SGB	St George Bank Ltd	Banking and Financial Services	N.A.
SGP	Stockland Ltd	Development, Construction and Investment in Land and	N.A.

		Commercial Property	
STO	Santos Group Ltd	Exploration, Development, Mining, Transportation and Marketing of Petroleum and Gas Products	N.A.
TAH	Tabcorp Holdings Ltd	Provision of Gambling and Other Entertainment Services	N.A.
WBC	Westpac Banking Corporation Ltd	Banking and Financial Services	N.A.
WES	Wesfarmers Ltd	Diversified Industrial including Retail/Wholesale, Energy/Fuel, Mining, Electricity, Chemical and Fertilizers	N.A.
WOW	Woolworths Ltd	Food, General Merchandise and Specialty Retailing	N.A.
WPL	Woodside Petroleum Ltd	Exploration, Development, Mining, Transportation and Marketing of Petroleum and Gas Products	N.A.

Table 6 Descriptive Statistics of the Australian Returns

ASX Code	Mean	Sigma	Skew	Kurtosis	Max	Min
AGL	0.132E-03	0.084	-1.94	688.0	6.53	-7.43
AMC	-0.265E-04	0.101	-4.37	4270.0	17.40	-15.20
ANZ	0.143E-03	0.093	-4.06	1660.0	8.25	-11.50
BHP	0.617E-04	0.090	3.30	7380.0	21.20	-20.10
BIL	0.118E-03	0.077	-4.16	3460.0	12.90	-13.00
CBA	0.148E-03	0.116	-1.79	1260.0	9.20	-9.47
CCL	0.825E-05	0.084	0.74	571.0	8.00	-6.39
CSL	0.254E-03	0.127	-0.44	821.0	10.50	-9.97
FGL	0.999E-04	0.136	-0.35	1600.0	18.00	-15.00
FXJ	0.388E-04	0.123	-1.28	806.0	9.00	-11.00
LLC	0.571E-04	0.066	-4.47	711.0	3.80	-6.81
NAB	0.106E-03	0.129	-2.46	2200.0	13.90	-15.40
NCM	0.158E-03	0.170	1.32	737.0	15.70	-14.50
QAN	0.629E-04	0.179	2.14	1340.0	19.20	-14.20
QBE	0.123E-03	0.133	-1.34	1950.0	18.00	-20.10
SGB	0.149E-03	0.099	1.59	2710.0	14.00	-15.50
SGP	0.794E-04	0.129	-2.19	2130.0	15.20	-16.80
STO	0.122E-03	0.125	-0.59	1250.0	11.00	-11.80
TAH	0.155E-03	0.108	-1.87	741.0	8.50	-8.72
WBC	0.144E-03	0.118	-0.31	1210.0	9.87	-9.53
WES	0.161E-03	0.111	0.42	725.0	9.33	-9.05
WOW	0.178E-03	0.140	-2.73	3930.0	21.10	-20.90
WPL	0.187E-03	0.120	0.71	1380.0	12.10	-10.50

Table 7 Descriptive Statistics after 10% Trimming of the Australian Returns

ASX Code	Mean	Sigma	Skew	Kurtosis	Max	Min
AGL	-0.170E-03	0.031	-0.059	1.93	0.075	-0.077
AMC	-0.449E-04	0.036	-0.013	2.01	0.090	-0.090
ANZ	0.163E-03	0.034	-0.004	0.79	0.077	-0.077
BHP	0.191E-04	0.026	0.008	0.70	0.059	-0.059
BIL	0.154E-03	0.017	0.080	2.63	0.045	-0.043
CBA	0.250E-03	0.029	0.018	0.76	0.065	-0.064
CCL	0.561E-04	0.027	0.001	2.47	0.068	-0.068
CSL	0.191E-03	0.033	0.035	2.33	0.084	-0.083
FGL	0.411E-04	0.031	0.008	3.91	0.087	-0.086
FXJ	-0.489E-04	0.038	-0.028	3.71	0.103	-0.104
LLC	0.556E-04	0.019	0.005	1.77	0.046	-0.046
NAB	0.149E-03	0.030	0.016	0.59	0.066	-0.065
NCM	0.183E-04	0.046	-0.005	2.43	0.117	-0.117
QAN	0.379E-04	0.040	0.017	3.84	0.110	-0.109
QBE	-0.164E-04	0.036	-0.030	2.10	0.090	-0.091
SGB	0.158E-03	0.029	0.027	1.90	0.071	-0.070
SGP	0.341E-03	0.015	0.456	5.43	0.046	-0.041
STO	-0.125E-04	0.043	-0.019	2.05	0.105	-0.106
TAH	0.865E-05	0.032	-0.014	2.04	0.080	-0.080
WBC	0.165E-03	0.033	0.014	0.97	0.077	-0.076
WES	-0.619E-04	0.026	-0.043	2.30	0.066	-0.068
WOW	0.299E-03	0.035	0.053	1.81	0.087	-0.085

WPL	-0.179E-03	0.034	-0.037	1.51	0.081	-0.082

Table 8 Classical Bonferroni (BF) '99 Percent Quantile' Linearity Test Statistics for the Australian Returns (2,920 Trispectrum Values)

ASX Code	10% BF - % of p-values in PD < 0.00003425	5% BF - % of p-values in PD < 0.00001712	1% BF - % of p-values in PD < 0.00000342
AGL	13.84%	13.49%	13.05%
AMC	14.90%	14.55%	14.21%
ANZ	7.53%	7.12%	7.02%
BHP	8.39%	7.81%	7.43%
BIL	13.90%	13.70%	13.42%
CBA	8.56%	8.29%	7.98%
CCL	16.20%	15.92%	15.68%
CSL	15.24%	14.90%	14.49%
FGL	19.83%	19.52%	19.32%
FXJ	18.90%	18.66%	18.42%
LLC	12.84%	12.47%	12.16%
NAB	6.30%	5.82%	5.72%
NCM	15.48%	15.27%	15.03%
QAN	18.46%	18.32%	18.05%
QBE	14.83%	14.21%	13.87%
SGB	12.64%	12.16%	12.05%
SGP	20.14%	19.76%	19.52%
STO	13.42%	13.12%	12.84%
TAH	14.38%	14.01 %	13.49%
WBC	7.40%	7.12 %	6.88%

WES	15.14%	14.69%	14.38%
WOW	12.98%	12.50%	12.26%
WPL	11.82%	11.37%	10.89%

Table 9 Classical Bonferroni (BF) Time Reversibility Test Statistics for the Australian Returns (2,920 Trispectrum Values)

ASX Code	10% BF - % of p-values in PD < 0.00003425	5% BF - % of p-values in PD < 0.00001712	1% BF - % of p-values in PD < 0.00000342
AGL	7.47%	6.44%	4.69%
AMC	9.38%	8.15%	5.99%
ANZ	6.27%	5.82%	4.62%
BHP	10.07%	8.97%	6.75%
BIL	25.58%	24.04%	21.61%
CBA	7.29%	6.13%	4.79%
CCL	11.03%	9.93%	7.57%
CSL	17.29%	15.72%	12.57%
FGL	21.13%	19.97%	16.88%
FXJ	16.85%	15.48%	12.88%
LLC	11.58%	10.03%	7.40%
NAB	9.08%	8.25%	6.68%
NCM	14.42%	12.67%	9.86%
QAN	23.15%	21.44%	18.25%
QBE	12.74%	11.47%	8.94%
SGB	8.77%	7.50%	5.48%
SGP	29.90%	28.42%	24.52%
STO	7.84%	6.92%	5.17%
TAH	8.87%	7.88%	6.03%

WBC	7.74%	6.82 %	5.07%
WES	11.51%	9.97%	7.05%
WOW	10.14%	8.77%	6.64%
WPL	10.62%	9.38%	7.36%

Table 10 Linearity and Time Reversibility 'CUSUM' Test p - Values

Stock Code	Trispectrum Time Reversibility Test p-values	Trispectrum Quantitle Nonlinearity Test p-values[16]
BIR	0.00000	0.00223
EOG	0.00000	0.07337
FE	0.00000	0.01548
IMN	0.00000	0.00000
AGL	0.00000	0.00000
AMC	0.00000	0.00000
ANZ	0.00000	0.00000
BHP	0.00000	0.00000
BIL	0.00000	0.00000
CBA	0.00000	0.00000
CCL	0.00000	0.00000
CSL	0.00000	0.00000
FGL	0.00000	0.00000

[16] Recall that the nonlinearity test statistics for the first four (NYSE) stock returns is calculated at the 90% Quantile. The nonlinearity test statistics is calculated at the 99% Quantile for the remaining (Australian) stock returns.

FXJ	0.00000	0.00000
LLC	0.00000	0.00000
NAB	0.00000	0.00000
NCM	0.00000	0.00000
QAN	0.00000	0.00000
QBE	0.00000	0.00000
SGB	0.00000	0.00000
SGP	0.00000	0.00000
STO	0.00000	0.00000
TAH	0.00000	0.00000
WBC	0.00000	0.00000
WES	0.00000	0.00000
WOW	0.00000	0.00000
WPL	0.00000	0.00000